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Corrigendum

Corrigendum of "Similarity analysis in magnetohydrodynamics: Hall effects on free convection flow and mass transfer past a semi-infinite vertical flat plate" [International Journal of Non-Linear Mechanics 38 (2003) 513–520]

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ABSTRACT

The problem of steady, laminar mixed convection heat and mass transfer past a semi-infinite vertical plate in the presence of Hall current has been studied. The governing partial differential equations describing the problem are transformed to a system of non-linear ordinary differential equations with appropriate boundary conditions using Lie's method of infinitesimal transformation groups. The non-linear ordinary differential equations are solved numerically using Chebyshev spectral method. The effects of various parameters on the velocity profiles, temperature and concentration profiles are presented and discussed. This work is an extension and correction for the paper by Megahed et al. [1], published in International Journal of Non-Linear Mechanics 38 (2003) 513–520.

1. Introduction

In recent years, several studies dealing with forced convection, free convection and mixed convection flow with simultaneous heat and mass transfer because of its natural processes, engineering and geophysical applications such as evaporation from a body of water with or without wind, the chemical vapor deposition of solid layers and the cooling of an air stream by evaporation, geothermal reservoirs, thermal insulation, cooling of nuclear reactors and underground energy transport.

Free convection flow occurs frequently in nature, it occurs not only due to temperature difference, but also due to concentration difference or combination of these two. Forced convection is a mechanism, or type of heat transport in which fluid motion is generated by an external source, the combined mode of free and forced convection is commonly called as mixed convection. Many authors have investigated the problems of the influence of free/mixed convection on a heat and mass transfer flow over a flat plate under different conditions [2–7].

In all the above investigations the electrical conductivity of the fluid was assumed to be uniform and low magnetic field intensity. In ionized gases the electric current is generally carried by electrons which undergo successive collisions with other charged or neutral particles and the current is not proportional to the applied potential except when the electric field is very weak. However, in the presence of strong electric field, the electrical conductivity is affected by the magnetic field. Consequently, the conductivity parallel to the electric field is reduced due to the gyration and drift of charged particles. Hence the current is reduced in the direction normal to both electric and magnetic fields. This phenomenon is known as the Hall effect, which can be taken into account within the range of MHD approximations. The study of magnetohydrodynamic flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators, flight magnetohydrodynamics, also it has important applications in many astrophysical situations as well as in Laboratory plasma [8–12].

On the other hand, it is well known that the Lie group method which discovered by the Norwegian mathematician Sophus Lie [13] at the beginning of the nineteenth century has played a vital role in the investigations of different mathematical aspects of solution systems governed by continuous equations during the past few decades. The primary objective of the Lie group method advanced by Sophus Lie is to fined one- or several-parameter local continuous transformation leaving the equations invariant and then exploit them to obtain the so called invariant similarity solutions, invariants, integrals of motion, etc. [14–19]. Applications of Lie group analysis for steady or unsteady two-dimensional problems may be found in Refs. [19–24].

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Nomenclature

$\vec{B}$	magnetic induction vector
B_0	imposed magnetic field
C	species concentration
C_p	specific heat at constant pressure
D	diffusion coefficient
e	electric charge
$\vec{E}$	electric field vector
$\vec{g}$	acceleration due to gravity
Gc	modified Grashof number $=g\beta^* C_0 L^3/\nu^2$
Gr	Grashof number $=g\beta T_0 L^3/\nu^2$
$\vec{H}$	magnetic field vector
$\vec{j}$	electric current density vector
k	thermal conductivity
L	dimensionless length
m	Hall parameter $=\sigma B_0/e\ n_e$
M	magnetic parameter $=\sigma B_0^2/\rho U_\infty$
N	relative buoyancy parameter $=Gr/Gc$
n_e	number density of electrons
P	fluid pressure
Pr	Prandtl number $=\rho\nu C_p/k$
Re	Reynolds number
Sc	Schmidt number $=\nu/D$
r_1, r_2	constants
T	temperature
U_∞	free stream velocity
$\vec{V}$	velocity vector
(u,v,w)	velocity component along (x,y,z)-axis
(x,y,z)	Cartesian coordinates

Greek symbols

β	volumetric expansion coefficient for heat transfer
β^*	volumetric expansion coefficient for mass transfer
η	similarity variable
θ	dimensionless temperature
φ	dimensionless concentration
μ_e	magnetic permeability
ν	kinematic viscosity
ρ	density
σ	electric conductivity
ψ	free stream function
λ	buoyancy or mixed convection parameter $=Gr/Re^2$

Subscripts

w	wall condition
∞	free stream condition
0	constant condition

Superscripts

$'$	differentiation with respect to η

Megahed et al. [1] obtained the similarity solutions for the effects of Hall current on a steady free convection flow and mass transfer past a semi-infinite vertical plate in a viscous incompressible electrically conducting fluid using a scaling group of transformations. Unfortunately, there are some errors founded in this paper. The main objective of this work is to extend and improve the work of Megahed et al. [1] taken into account the errors mentioned above, with considering the presence of free stream velocity, variable wall temperature and variable wall concentration. All errors have been stated in Section 3 in details. We apply the symmetry group method to reduce the system of PDEs governing two-dimensional steady, laminar, hydromagnetic mixed convection flow of mass transfer of an incompressible, viscous and electrically conducting fluid past a semi-infinite vertical flat plate to non-linear ODEs. The reduced non-linear ODEs solved numerically using Chebyshev spectral method for various parameters.

2. Mathematical analysis

Consider a steady, two-dimensional, laminar flow, heat and mass transfer of an incompressible, viscous and electrically conducting fluid past a semi-infinite vertical flat plate. The X-axis is taken along the direction of motion, the Y-axis is taken normal to the surface and Z-axis is transverse to the XY-plane (see Fig. 1).

It is assumed that a non-uniform magnetic field B is applied in the Y-direction, it is also assumed that the surface temperature taken as a function of the distance $T_w(x/L)$, while the ambient fluid has a uniform temperature T_∞, where $T_w(x/L) > T_\infty$ corresponds to a heated plate and $T_w(x/L) < T_\infty$ corresponds to a cooled plate.

In studying the Hall effect, the magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected in comparison with an applied field. The fundamental equations with generalized Ohm's law and Maxwell's equations under the assumption that the fluid is non-magnetic, neglecting the ion slip, thermoelectric effect as well as viscous and electrical dissipation together with the short circuit condition have the form

$$\nabla.\vec{V} = 0, \tag{1}$$

$$(\vec{V}.\nabla)\vec{V} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\vec{V} + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) + \frac{1}{\rho}\vec{j}\times\vec{B}, \tag{2}$$

$$(\vec{V}.\nabla)T = \left(\frac{k}{\rho C_p}\right)\nabla^2 T, \tag{3}$$

$$(\vec{V}.\nabla)C = D\nabla^2 C, \tag{4}$$

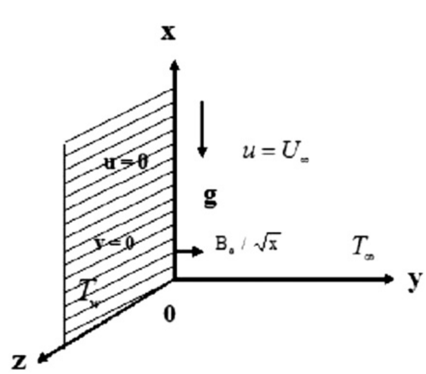

Fig. 1. Physical model and coordinates.

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$$\vec{j}=\sigma\left[E+V\times\vec{B}-\frac{1}{en_e}\vec{j}\times\vec{B}\right], \tag{5}$$

$$\nabla.\vec{B}=0, \tag{6a}$$

$$\nabla\times\vec{H}=\vec{j}, \tag{6b}$$

$$\nabla\times\vec{E}=0, \tag{6c}$$

where $B=(0,B_y(x/L),0)$. To simplify the problem, we assume that there is no variation of flow or heat and mass transfer quantities in Z-direction, the equation of conservation of electric charge $\nabla.\vec{j}=0$ gives j_y=constant, where $\vec{j}=(j_x j_y j_z)$. It is also assumed that the plate is non-conducting. This implies $j_y=0$ at the plate and hence zero everywhere. In the absence of an externally applied electric field and with negligible effects of polarization of the ionized gas, we also assume that E=0. Under the above assumptions and usual Boussinesq's approximation, the boundary layer equations of mass, momentum, energy and species concentration for the present model can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma(B_y)^2}{\rho(1+m^2)}((u-U_\infty)+mw), \tag{8}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\nu\frac{\partial^2 w}{\partial y^2}-\frac{\sigma(B_y)^2}{\rho(1+m^2)}(w-m(u-U_\infty)), \tag{9}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}, \tag{10}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}, \tag{11}$$

subject to the boundary conditions

$$y=0: u=v=w=0,\quad T\left(\frac{x}{L}\right)=T_w-T_\infty=T_0\left(\frac{x}{L}\right)^{r_1},\quad C(x)=C_w-C_\infty=C_0\left(\frac{x}{L}\right)^{r_2}, \tag{12a}$$

$$y\to\infty: u=U_\infty,\quad w=0,\quad T=T_\infty,\quad C=C_\infty, \tag{12b}$$

we notice that for $T_0>0$ the plate is heated and for $T_0<0$ the plate is cooled. Here we introduce the non-dimensional variables

$$\overline{x}=\frac{x}{L},\quad \overline{y}=\frac{y}{L}\left(\frac{U_\infty L}{\nu}\right)^{1/2},\quad \overline{u}=\frac{u}{U_\infty},\quad \overline{v}=\frac{v}{U_\infty}\left(\frac{U_\infty L}{\nu}\right)^{1/2},$$

$$\overline{w}=\frac{w}{U_\infty},\quad \theta=\frac{T-T_\infty}{T_0},\quad \varphi=\frac{C-C_\infty}{C_0}, \tag{13}$$

also we define ψ as: $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$, then Eqs. (7)–(12) will have the form (dropping the bar mark for convenience)

$$\psi_y\psi_{xy}-\psi_x\psi_{yy}-\psi_{yyy}-\lambda(\theta+N\varphi)+\frac{\sigma(B_y)^2}{\rho(1+m^2)}(\psi_y+mw-1)=0, \tag{14}$$

$$\psi_y w_x-\psi_x w_y-w_{yy}+\frac{\sigma(B_y)^2}{\rho(1+m^2)}(w-m(\psi_y-1))=0, \tag{15}$$

$$\psi_y\theta_x-\psi_x\theta_y-\frac{1}{\mathrm{Pr}}\theta_{yy}=0, \tag{16}$$

$$\psi_y\varphi_x-\psi_x\varphi_y-\frac{1}{Sc}\varphi_{yy}=0, \tag{17}$$

with the boundary conditions

$$y=0: \psi_y=\psi_x=w=0,\quad \theta=x^{r_1},\quad \varphi=x^{r_2}, \tag{18a}$$

$$y\to\infty: \psi_y=1,\quad w=\theta=\varphi=0, \tag{18b}$$

where $\lambda=Gr/Re^2$ is the mixed convection parameter, $N=Gr/Gc$ is the Relative buoyancy parameter, $\mathrm{Pr}=\rho\nu C_p/k$ is the Prandtl number, $Sc=\nu/D$ is the Schmidt number, σ is the electric conductivity, ρ is the density and m is the Hall parameter. In next section, we will display the errors which have been founded in Megahed et al. [1]

3. Remarks

In this section, it should have been indicated to all the errors which have been founded in Megahed et al. [1], before applying the group of transformation.

First, authors defined U_∞ as a free stream velocity although, they solved a problem of free convection flow ($U_\infty=0$), then the dimensionless quantities defined in (Eq. (19) in their paper)

$$\overline{x}=\frac{xU_\infty}{\nu},\quad \overline{y}=\frac{yU_\infty}{\nu},\quad \overline{u}=\frac{u}{U_\infty},\quad \overline{v}=\frac{v}{U_\infty},\quad \overline{w}=\frac{w}{U_\infty},$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\quad \varphi=\frac{C-C_\infty}{C_w-C_\infty}, \tag{19}$$

therefore some of previous quantities will vanish and other will tend to infinity.

Second, for free convection problem authors used scaling group of transformation (Eq. (26) in their paper)

$$x^*=\lambda^{c_1}x,\quad y^*=\lambda^{c_2}y,\quad u^*=\lambda^{c_3}u,\quad v^*=\lambda^{c_4}v,\quad w^*=\lambda^{c_5}w,$$

$$\theta^*=\lambda^{c_6}\theta,\quad \varphi^*=\lambda^{c_7}\varphi, \tag{20}$$

authors substitute from this transformation into the basic equations, they get (Eq. (32) in their paper)

$$c_2=c_1,\quad c_3=c_4=c_5=-c_1,\quad c_6=c_7=-3c_1, \tag{21}$$

they obtained the similarity variable and similarity functions (Eq. (34) in their paper) as

$$\eta=\frac{y}{x},\quad u=\frac{F_1(\eta)}{x},\quad v=\frac{F_2(\eta)}{x},\quad w=\frac{F_3(\eta)}{x},$$

$$\theta=\frac{F_4(\eta)}{x^3},\quad \varphi=\frac{F_5(\eta)}{x^3}, \tag{22}$$

but they ignored to satisfy from the transformation into the boundary conditions. Satisfying in the boundary conditions (Eq. (25a) in their paper), we find

$$\theta^*\lambda^{-c_6}=1,\quad \varphi^*\lambda^{-c_7}=1, \tag{23}$$

only one solution is obtained from Eq. (23), that is

$$c_6=c_7=0, \tag{24}$$

which leads to $c_1=0$, and then the similarity variable and the similarity functions defined in (Eq. (34) in their paper) are incorrect.

Third, however, substituting from Eq. (22) into the boundary conditions (Eq. (25a) in their paper), we obtain

$$\eta=0: F_4=x^3,\quad F_5=x^3, \tag{25}$$

thus, Eq. (40a) in their paper is incorrect. All these errors inevitably lead to wrong results.

Fourth, as well as it is known in boundary layer theory that velocity, temperature and concentration profiles approach the ambient fluid conditions asymptotically [25]. It is shown in Figs. 2–9 (in their paper) that all velocities, temperature and concentration profiles do not approach the horizontal axis asymptotically and intersect it.

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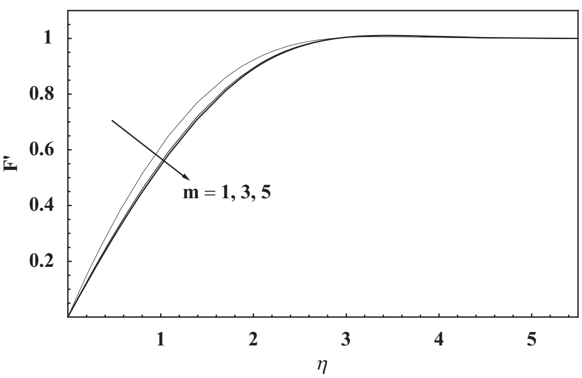

Fig. 2. Velocity distribution for various values of m with $M=0.5$, $Pr=0.72$, $Sc=0.22$, $\lambda=0.5$, $N=1$.

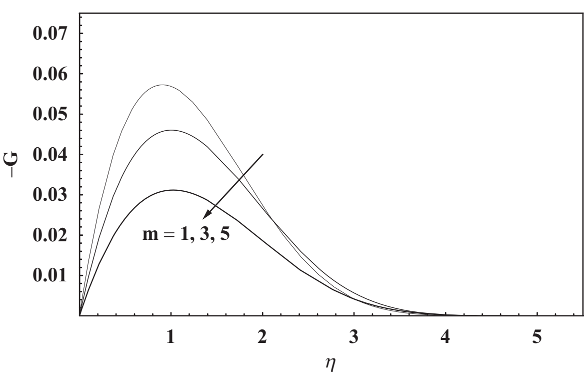

Fig. 3. Cross-velocity distribution for various values of m with $M=0.5$, $Pr=0.72$, $Sc=0.22$, $\lambda=0.5$, $N=1$.

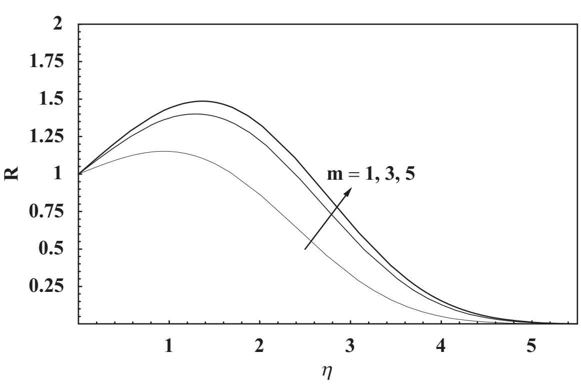

Fig. 4. Temperature distribution for various values of m with $M=0.5$, $Pr=0.72$, $Sc=0.22$, $\lambda=0.5$, $N=1$.

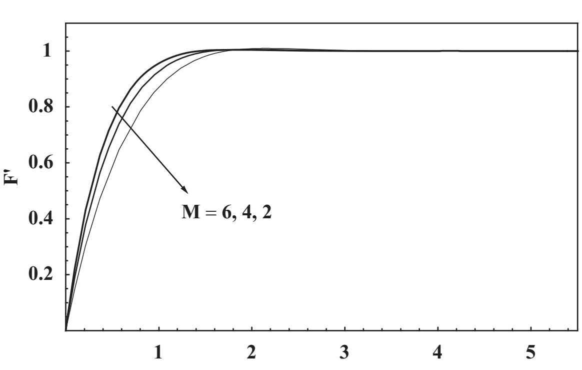

Fig. 5. Velocity distribution for various values of M with $m=0.1$, $Pr=0.72$, $Sc=0.22$, $\lambda=0.3$, $N=0.1$.

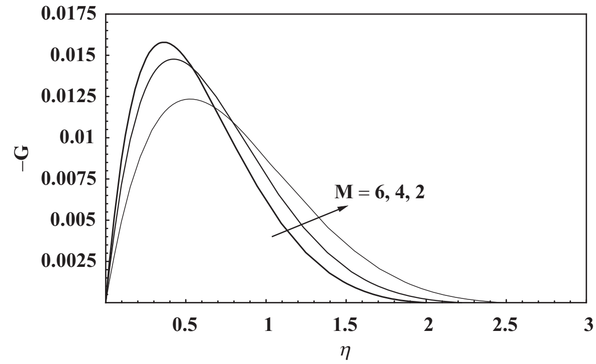

Fig. 6. Cross-velocity distribution for various values of M with $m=0.1$, $Pr=0.72$, $Sc=0.22$, $\lambda=0.3$, $N=0.1$.

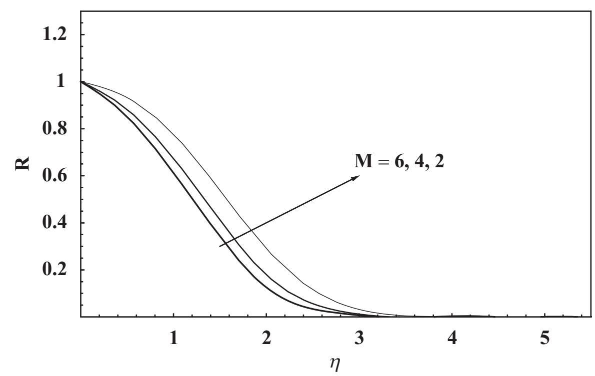

Fig. 7. Temperature distribution for various values of M with $m=0.1$, $Pr=0.72$, $Sc=0.22$, $\lambda=0.3$, $N=0.1$.

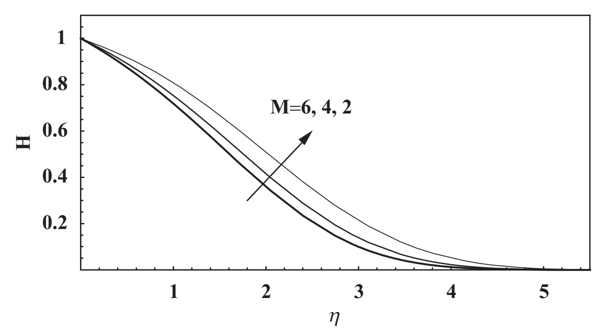

Fig. 8. Mass distribution for various values of M with $m=0.1$, $Pr=0.72$, $Sc=0.22$, $\lambda=0.3$, $N=0.1$.

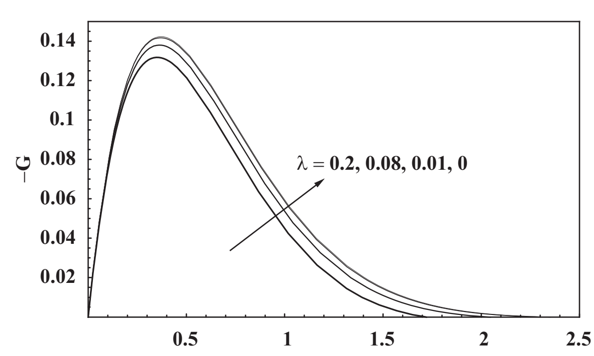

Fig. 9. Cross-velocity distribution for various values of λ with $m=1$, $M=10$, $Pr=0.72$, $Sc=0.22$, $N=0.5$.

4. Symmetry reductions

In order to determine a similarity solution for Eqs. (14)–(17), we seek a one parameter Lie group of transformation

$$\tilde{x} = x + \varepsilon\zeta_x + O(\varepsilon^2),$$

$$\tilde{y} = y + \varepsilon\zeta_y + O(\varepsilon^2),$$

$$\tilde{\psi} = \psi + \varepsilon\eta_\psi + O(\varepsilon^2),$$

$$\tilde{w} = w + \varepsilon\eta_w + O(\varepsilon^2),$$

$$\tilde{\theta} = \theta + \varepsilon\eta_\theta + O(\varepsilon^2),$$

$$\tilde{\varphi} = \varphi + \varepsilon\eta_\varphi + O(\varepsilon^2),$$

$$\tilde{B} = B + \varepsilon\eta_B + O(\varepsilon^2), \quad (26)$$

where ε is a small parameter and ζ_x, ζ_y, η_ψ, η_w, η_θ, η_ϑ and η_B are infinitesimal element of the group. We search for functions ζ_x, ζ_y, η_ψ, η_w, η_θ, η_ϑ and η_B such that the system of Eqs. (14)–(17) is invariant under the transformation (26) up to first order in ε. Following the usual procedure outlined in [13–18], the invariance of the system (14)–(17) under the infinitesimal group of transformations (26) yields the following forms of the infinitesimals

$$\zeta_x = -2c_1x + c_4, \quad \zeta_y = -c_1y + f_1(x), \quad \eta_\psi = c_2 - c_1\psi,$$

$$\eta_w = 0, \quad \eta_\theta = 2c_1\theta + \frac{N}{N-1}c_3, \quad \eta_\varphi = 2c_1\varphi - \frac{1}{N-1}c_3, \quad \eta_{B_y} = c_1B_y \quad (27)$$

where c_1, c_2, c_3 and c_4 are constants. The invariance of the boundary conditions (18) under the transformation (26) gives that

$$f_1(x) = c_2 = c_3 = 0 = c_4 = 0, \quad r_1 = r_2 = -1. \quad (28)$$

Similarity reduction solutions can be obtained from the characteristic equation

$$\frac{dx}{-2x} = \frac{dy}{-y} = \frac{d\psi}{-\psi} = \frac{dw}{0} = \frac{d\theta}{2\theta} = \frac{d\varphi}{2\varphi} = \frac{dB_y}{B_y}, \quad (29)$$

then the similarity transformations of this group are

$$\eta = \frac{y}{\sqrt{x}}, \quad \psi = \sqrt{x}F(\eta), \quad w = G(\eta), \quad \theta = \frac{R(\eta)}{x},$$

$$\varphi = \frac{H(\eta)}{x}, \quad B_y = \frac{B_0}{\sqrt{x}}, \quad (30)$$

Substituting (30) into Eqs. (14)–(18), we finally obtain a system of non-linear ordinary differential equations

$$F''' + \frac{1}{2}FF'' + \lambda(R + N/H) - \frac{M}{(1+m^2)}(F' + mG - 1) = 0, \quad (31)$$

$$G'' + \frac{1}{2}G'F - \frac{M}{(1+m^2)}(G - mF' + m) = 0, \quad (32)$$

$$\frac{1}{\Pr}R'' + RF' + \frac{1}{2}FR' = 0, \quad (33)$$

$$\frac{1}{Sc}H'' + HF' + \frac{1}{2}FH' = 0, \quad (34)$$

with appropriate boundary conditions

$$\eta = 0: F = F' = 0, \quad G = 0, \quad H = R = 1, \quad (35a)$$

$$\eta \to \infty: F' = 1, \quad G = R = H = 0. \quad (35b)$$

5. Results and discussion

The non-linear ordinary differential Eqs. (31)–(34) with boundary conditions (35) solved numerically by using Chebyshev method [26]. In order to get physical insight into the problem, the velocities fields $F'(\eta)$, $-G(\eta)$, temperature $R(\eta)$ and concentration field $H(\eta)$ have been discussed by assigning numerical values of Hall parameter m, magnetic field parameter M, buoyancy parameter λ, rational parameter N and Schmidt number Sc. The computational work has been carried out by taking the Prandtl number Pr equals 0.72 which represents air at 20 °C and one atmospheric pressure. We choose different values of Schmidt number describing various gases like hydrogen ($Sc = 0.22$), helium ($Sc = 0.30$), water vapor ($Sc = 0.60$), oxygen ($Sc = 0.66$), and ammonia ($Sc = 0.78$). The values of buoyancy parameter λ, rational parameter N are taken to be ($\lambda > 0$), ($N > 0$) corresponding to a assisting flow (heated plate). The numerical results obtained are illustrated in Figs. 2–14, the numerical values of the physical parameters m, M, λ, N, Sc are listed in the figure labels.

The effects of Hall parameter m on the axial velocity F', cross-velocity $-G(\eta)$ and temperature profiles are illustrated Figs. 2–4. It is evident that the effect of increasing m is to decrease the axial velocity F' and the cross-velocity $-G(\eta)$ while it increases the temperature field.

Figs. 5–8 show the velocities, temperature and concentration profiles in the boundary layer for various values of the magnetic parameter M. Fig. 5 depicts that, increase in the value of magnetic parameter, M, leads to an increase in the velocity profile within the boundary layer, and the thickness of boundary layer increases with decrease in the value of M. This is because Lorentz force arising due to magnetic field acts as an accelerating force in

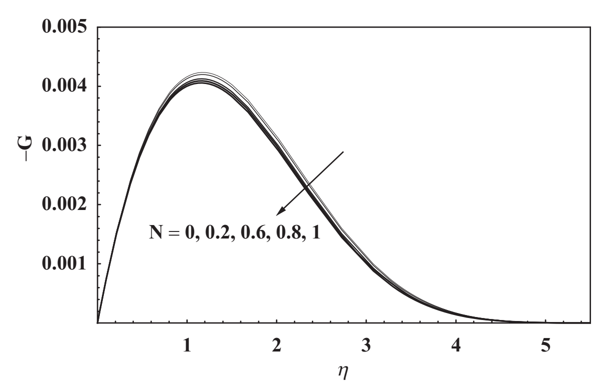

Fig. 10. Cross-velocity distribution for various values of N with $m = 0.1$, $M = 0.1$, $Pr = 0.72$, $Sc = 0.22$, $\lambda = 0.01$.

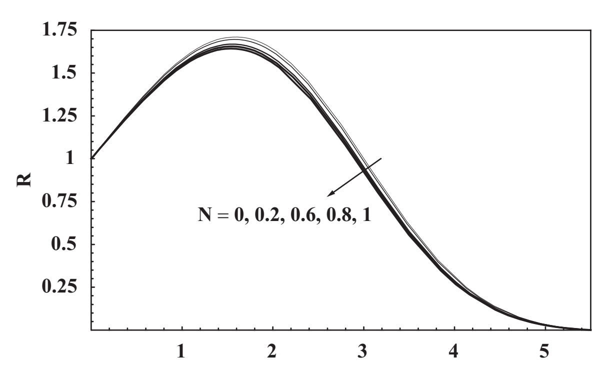

Fig. 11. Temperature distribution for various values of N with $m = 0.1$, $M = 0.1$, $Pr = 0.72$, $Sc = 0.22$, $\lambda = 0.01$.

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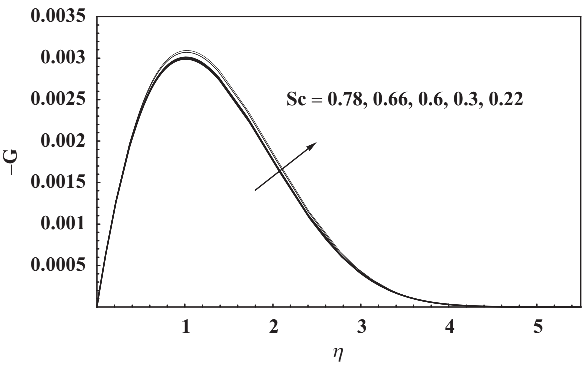

Fig. 12. Cross-velocity distribution for various values of *Sc* with $m=0.1$, $M=0.1$, $Pr=0.72$, $\lambda=0.04$, $N=1$.

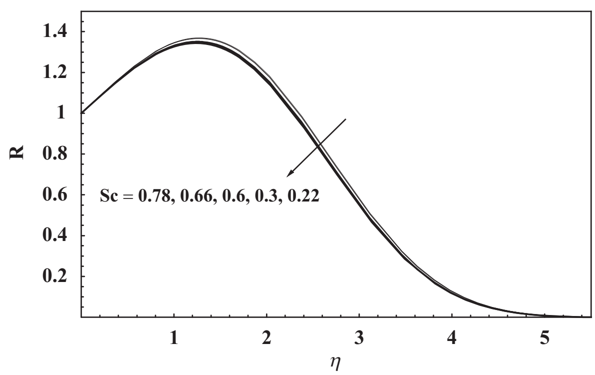

Fig. 13. Temperature distribution for various values of *Sc* with $m=0.1$, $M=0.1$, $Pr=0.72$, $\lambda=0.04$, $N=1$.

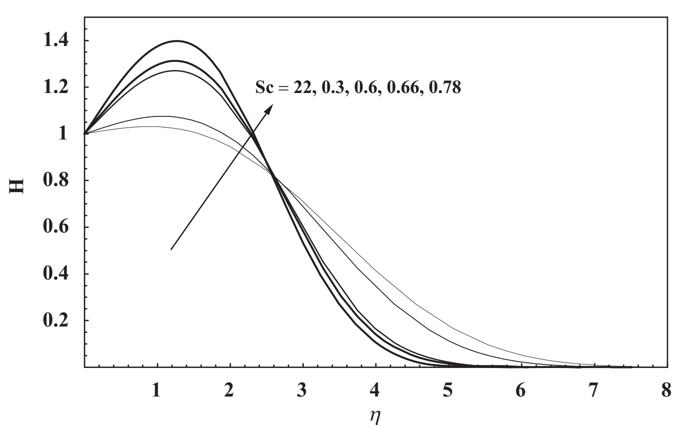

Fig. 14. Mass distribution for various values of *Sc* with $m=0.1$, $M=0.1$, $Pr=0.72$, $\lambda=0.04$, $N=1$.

reducing frictional resistance which results in higher values of velocity profiles. From Fig. 6 one observes that increase in the value of magnetic parameter, M, leads to an increase in the cross velocity $-G$ near the surface and the reverse is true far away from the surface. Fig. 7 displays the variation of temperature profiles for various values of magnetic parameter M, from this figure it is evident that the temperature profile decreases at a faster rate for all values of magnetic parameter M within the boundary layer, this is due to the fact that the applied magnetic field produces a body force which is responsible for reducing the temperature in the boundary layer. This shows the fact that the rate of cooling is faster for large values of M. Fig. 8 shows that the mass concentration is decreasing with increasing M.

In the case of fixed values for Hall parameter, magnetic parameter and Schmidt number, it is observed that the increase of mixed convection parameter accelerates the fluid motion in the normal direction which are vivid through Fig. 9.

Figs. 10 and 11 describe the effect of the rational parameter N on the cross velocity $-G$ and the temperature distribution. These figures indicate that the cross-velocity and the temperature decreases as N increases.

Figs. 12–14 present the behavior of the cross-velocity, temperature and concentration profiles for the variation of Schmidt number Sc. It is notice that the velocity $-G(\eta)$ increases and the temperature decreases R as the Schmidt number decreases. From Fig. 14 it seen that the effect of decreasing the Schmidt number Sc is to decrease the concentration H greatly near the plate whereas the situation is reversed far away from the plat, i.e., the decrease in the values of Sc increase the concentration as one moves out toward the boundary layer.

6. Conclusion

The problem of a steady, laminar, hydromagnetic heat and mass transfer along a non-isothermal vertical plate taken in the presence of the Hall effect was studied. The Lie symmetry analysis has been applied on the governing system of partial differential equations which has transformed to a system of ordinary differential equations depend on six dimensionless parameters, namely a Prandtl number Pr Hall parameter m, the magnetic parameter M, buoyancy parameter λ, rational parameter N, and Schmidt number Sc. A numerical study for the transformed non-linear ODEs were solved Chebyshev method. The effects of these physical parameters on the axial velocity $F'(\eta)$, the cross velocity profile $-G(\eta)$, temperature profiles $R(\eta)$, and the mass profiles $H(\eta)$ are illustrated graphically. It is found that the axial velocity F' decreases with increasing of Hall parameter, and increases with of magnetic parameter. On the other hand the cross velocity $-G$ decreases with the increasing of Hall parameter, magnetic parameter, buoyancy parameter, rational parameter and Schmidt number.

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